SADLER UNIT 3 CHAPTER 3

EXERCISE 3A

Q1. $f(x) = x+1$

$g(x) = 2x-3$

a) $g(f(x)) = 2(x+1)-3$
$= 2x-1$

$g(f(\{0,1,2,3,4\})) = \{-1,1,3,5,7\}$

b) $f(g(x)) = (2x-3)+1$
$= 2x-2$

$f(g(\{0,1,2,3,4\})) = \{-2,0,2,4,6\}$

c) $g(g(x)) = 2(2x-3)-3$
$= 4x-9$

$g(g(\{0,1,2,3,4\})) = \{-9,-5,-1,3,7\}$

Q2. $f(x) = x+3$

$g(x) = (x-1)^2$

$h(x) = x^3$

a) $g(f(x)) = (x+3-1)^2$
$= (x+2)^2$

$g(f(\{1,2,3\})) = \{9,16,25\}$

b) $f(g(h(x))) = (x^3-1)^2+3$

$f(g(h(\{1,2,3\}))) = \{3,52,679\}$

c) $h(g(f(x))) = [(x+3-1)^2]^3$
$= (x+2)^6$

$h(g(f(\{1,2,3\}))) = \{729,4096,15625\}$.

Q3. $f(x) = x+5$

$g(x) = x-5$

a) $D_f = \{x : x \in \mathbb{R}\}$

$R_f = \{y : y \in \mathbb{R}\}$

b) $D_g = \{x : x \in \mathbb{R}\}$

$R_g = \{y : y \in \mathbb{R}\}$

c) $f(x)+g(x) = 2x$

$D_{f+g} = \{x : x \in \mathbb{R}\}$

$R_{f+g} = \{y : y \in \mathbb{R}\}$.

d) $f(x)-g(x) = 10$

$D_{f-g} = \{x : x \in \mathbb{R}\}$

$R_{f-g} = \{y : y \in \mathbb{R}, y=10\}$.

e) $f(x)\cdot g(x) = (x+5)(x-5)$
$= x^2-25$

$D_{fg} = \{x ; x \in \mathbb{R}\}$

$R_{fg} = \{y : y \in \mathbb{R}, y \geq -25\}$.

f) $\dfrac{f(x)}{g(x)} = \dfrac{x+5}{x-5} \Rightarrow \dfrac{1(x-5)+10}{x-5}$

$D_{\frac{f}{g}} = \{x ; x \in \mathbb{R}, x \neq 5\}$

$R_{\frac{f}{g}} = \{y : y \in \mathbb{R}, y \neq 1\}$.

Q4. $f(x) = 3x+2$, $g(x) = \dfrac{2}{x}$, $h(x) = \sqrt{x}$

a) $\dfrac{2}{3x+2} = g(f(x))$.

b) $\sqrt{3x+2} = h(f(x))$.

c) $\dfrac{6}{x} + 2 = f(g(x))$

d) $3\sqrt{x} + 2 = f(h(x))$.

e) $\dfrac{2}{\sqrt{x}} = g(h(x))$.

f) $\sqrt{\dfrac{2}{x}} = h(g(x))$.

g) $9x+8 = f(f(x))$.

h) $x^{0.25} = h(h(x))$.

i) $27x+26 = f(f(f(x)))$.

Q5. $f(x) = 2x-3$, $g(x) = 4x+1$, $h(x) = x^2+1$

a) $f(f(x)) = 2(2x-3)-3$
$= 4x-6-3$
$= 4x-9$.

b) $g(g(x)) = 4(4x+1)+1$
$= 16x+4+1$
$= 16x+5$

c) $h(h(x)) = (x^2+1)^2+1$
$= x^4+2x^2+1+1$
$= x^4+2x^2+2$

d) $f(g(x)) = 2(4x+1)-3$
$= 8x+2-3$
$= 8x-1$

e) $g(f(x)) = 4(2x-3)+1$
$= 8x - 12 + 1$
$= 8x - 11$

f) $f(h(x)) = 2(x^2+1) - 3$
$= 2x^2 + 2 - 3$
$= 2x^2 - 1$

g) $h(f(x)) = (2x-3)^2 + 1$
$= 4x^2 - 2(6x) + 9 + 1$
$= 4x^2 - 12x + 10$

h) $g(h(x)) = 4(x^2+1)+1$
$= 4x^2 + 4 + 1$
$= 4x^2 + 5$

i) $h(g(x)) = (4x+1)^2 + 1$
$= 16x^2 + 8x + 1 + 1$
$= 16x^2 + 8x + 2$

Q6 $f(x) = 2x+5$
$g(x) = 3x+1$
$h(x) = 1 + \frac{2}{x}$

a) $f(f(x)) = 2(2x+5)+5$
$= 4x + 10 + 5$
$= 4x + 15$

b) $g(g(x)) = 3(3x+1)+1$
$= 9x + 3 + 1$
$= 9x + 4$

c) $h(h(x)) = 1 + \frac{2}{1+\frac{2}{x}}$
$= 1 + \frac{2}{\frac{x+2}{x}}$
$= 1 + \frac{2x}{x+2}$

[or $= \frac{x+2+2x}{x+2}$
$= \frac{3x+2}{x+2}$]

d) $f(g(x)) = 2(3x+1)+5$
$= 6x + 2 + 5$
$= \underline{6x + 7}$

e) $g(f(x)) = 3(2x+5)+1$
$= 6x + 15 + 1$
$= 6x + 16$

f) $f(h(x)) = 2\left(1 + \frac{2}{x}\right) + 5$
$= 2 + \frac{4}{x} + 5$
$= 7 + \frac{4}{x}$

g) $h(f(x)) = 1 + \frac{2}{2x+5}$

[or $= \frac{2x+5+2}{2x+5}$
$= \frac{2x+7}{2x+5}$]

h) $g(h(x)) = 3\left(1 + \frac{2}{x}\right) + 1$
$= 3 + \frac{6}{x} + 1$
$= 4 + \frac{6}{x}$

i) $h(g(x)) = 1 + \frac{2}{3x+1}$
$= \frac{3x+1+2}{3x+1}$
$= \frac{3x+3}{3x+1}$

Q7. $g(f(x)) = \sqrt{x-4}$
$D_f = \{x : x \in \mathbb{R}, x \geq 4\}$

Q8 $g(f(x)) = \sqrt{4-x}$
$D_f = \{x : x \in \mathbb{R}, x \leq 4\}$

Q9. $g(f(x)) = \sqrt{4-x^2}$
$D_f = \{x : x \in \mathbb{R}, -2 \leq x \leq 2\}$

Q10. $g(f(x)) = \sqrt{4-|x|}$
$D_f = \{x : x \in \mathbb{R}, -4 \leq x \leq 4\}$

Q11. $g(f(x)) = \sqrt{x+3-5}$

$= \sqrt{x-2}$

$D_f = \{x : x \in \mathbb{R}, x \geq 2\}$

Q12. $g(f(x)) = \sqrt{x-6+3}$

$= \sqrt{x-3}$

$D_f = \{x : x \in \mathbb{R}, x \geq 3\}$

Q13. $f(x) = x^2 + 3$

$g(x) = \frac{1}{x}$

a) $f(3) = 9 + 3$

$= \underline{\underline{12}}$

b) $f(-3) = 9 + 3$

$= \underline{\underline{12}}$

c) $g(2) = \underline{\underline{\frac{1}{2}}}$

d) $f(g(x)) = \frac{1}{x^2} + 3$

$f(g(1)) = 1 + 3$

$= \underline{\underline{4}}$

e) $g(f(x)) = \frac{1}{x^2+3}$

$g(f(1)) = \frac{1}{1+3}$

$= \underline{\underline{\frac{1}{4}}}$

f) $D_f = \{x : x \in \mathbb{R}\}$

$R_f = \{y : y \in \mathbb{R}, y \geq 3\}$

g) $D_g = \{x : x \in \mathbb{R}, x \neq 0\}$

$R_g = \{y : y \in \mathbb{R}, y \neq 0\}$

h) $D_{gf} = \{x : x \in \mathbb{R}\}$

$R_{gf} = \{y : y \in \mathbb{R}, 0 \leq y \leq \frac{1}{3}\}$

(*) Minimum value of x^2+3 occurs when $x = 0$, $\therefore$ max value of $\frac{1}{x^2+3}$ occurs at $x = 0$.

i) $D_{fg} = \{x : x \in \mathbb{R}, x \neq 0\}$

$R_{fg} = \{y : y \in \mathbb{R}, y > 3\}$.

Q14 $f(x) = 25 - x^2$

$g(x) = \sqrt{x}$

a) $f(5) = 25 - 25$

$= \underline{\underline{0}}$

b) $f(-5) = 25 - 25$

$= \underline{\underline{0}}$

c) $g(4) = \sqrt{4}$

$= \underline{\underline{2}}$

d) $f(g(x)) = 25 - (\sqrt{x})^2$

$= 25 - x$

$f(g(4)) = 25 - 4$

$= \underline{\underline{21}}$

e) $g(f(x)) = \sqrt{25 - x^2}$

$g(f(4)) = \sqrt{25 - 16}$

$= \sqrt{9}$

$= \underline{\underline{3}}$

f) $D_f = \{x : x \in \mathbb{R}\}$

$R_f = \{y : y \in \mathbb{R}, y \leq 25\}$

g) $D_g = \{x : x \in \mathbb{R}, x \geq 0\}$

$R_g = \{y : y \in \mathbb{R}, y \geq 0\}$.

h) $g(f(x)) = \sqrt{25 - x^2}$

$D_{gf} = \{x : x \in \mathbb{R}, -5 \leq x \leq 5\}$

$R_{gf} = \{y : y \in \mathbb{R}, 0 \leq y \leq 5\}$

(*) maximum value of $25 - x^2$ occurs at $x = 0$, $\therefore$ max value of gf will also occur at $x = 0$.

i) $f(g(x)) = 25 - x$

$x \geq 0 \rightarrow \boxed{g(x)} \rightarrow y \geq 0$

$\downarrow$

$x \in \mathbb{R}, x \geq 0 \rightarrow \boxed{f(g(x))} \rightarrow y \leq 25$

$\therefore D_{fg} = \{x : x \in \mathbb{R}, x \geq 0\}$

$R_{fg} = \{y : y \in \mathbb{R}, y \leq 25\}$.

Q15. $f(x) = x+2$

$g(x) = \dfrac{1}{x-3}$

a) $g \circ f(x) = \dfrac{1}{x-1}$

$x \in \mathbb{R} \rightarrow \boxed{f(x)} \rightarrow y \in \mathbb{R}$

$\downarrow$

$x \neq 3 \rightarrow \boxed{g(x)} \rightarrow \boxed{y \neq 0}$

$\downarrow$

$3 \neq x+2$

$\boxed{x \neq 1} \rightarrow \boxed{g(f(x))} \rightarrow \boxed{y \neq 0}$

$\therefore D_{gf} = \{x : x \in \mathbb{R}, x \neq 1\}$

$R_{gf} = \{y : y \in \mathbb{R}, y \neq 0\}$.

b) $f \circ g(x) = \dfrac{1}{x-3} + 2$

$\boxed{x \neq 3} \rightarrow \boxed{g(x)} \rightarrow y \neq 0$

$\downarrow$

$x \in \mathbb{R} \rightarrow \boxed{f(x)} \rightarrow y \in \mathbb{R}$

$\downarrow$

$x \neq 0 \rightarrow \boxed{f(g(x))} \rightarrow \boxed{y \neq 2}$

$\therefore D_{fg} = \{x : x \in \mathbb{R}, x \neq 3\}$

$R_{fg} = \{y : y \in \mathbb{R}, y \neq 2\}$.

Q16. $f(x) = \sqrt{x}$

$g(x) = 2x-1$

a) $g \circ f(x) = 2\sqrt{x} - 1$

$x \geq 0 \rightarrow \boxed{f(x)} \rightarrow y \geq 0$

$\downarrow$

$x \in \mathbb{R} \rightarrow \boxed{g(x)} \rightarrow y \in \mathbb{R}$

$\downarrow$

$\boxed{x \geq 0} \rightarrow \boxed{g(f(x))} \rightarrow \boxed{y \geq -1}$

$D_{gf} = \{x : x \in \mathbb{R}, x \geq 0\}$

$R_{gf} = \{y : y \in \mathbb{R}, y \geq -1\}$.

b) $f \circ g(x) = \sqrt{2x-1}$

$x \in \mathbb{R} \rightarrow \boxed{g(x)} \rightarrow y \in \mathbb{R}$

$\downarrow$

$x \geq 0 \rightarrow \boxed{f(x)} \rightarrow \boxed{y \geq 0}$

$\downarrow$

$2x - 1 \geq 0$

$\boxed{x \geq \tfrac{1}{2}} \rightarrow \boxed{f(g(x))} \rightarrow \boxed{y \geq 0}$

$D_{fg} = \{x : x \in \mathbb{R}, x \geq \tfrac{1}{2}\}$

$R_{fg} = \{y : y \in \mathbb{R}, y \geq 0\}$.

Q17. $f(x) = \dfrac{1}{x^2}$

$g(x) = \sqrt{x}$

a) $g \circ f(x) = \dfrac{1}{x}$

$\boxed{x \neq 0} \rightarrow \boxed{f(x)} \rightarrow y \neq 0$

$\downarrow$

$x \geq 0 \rightarrow \boxed{g(x)} \rightarrow y \geq 0$

$\downarrow$

$\dfrac{1}{x^2} \geq 0$

but $x \neq 0$

$\hookrightarrow \boxed{g(f(x))} \rightarrow \boxed{y > 0}$

$D_{gf} = \{x : x \in \mathbb{R}, x \neq 0\}$

$R_{gf} = \{y : y \in \mathbb{R}, y > 0\}$.

b) $f \circ g(x) = \dfrac{1}{x}$

$x \geq 0 \rightarrow \boxed{g(x)} \rightarrow y \geq 0$

$\downarrow$

$x \neq 0 \rightarrow \boxed{f(x)} \rightarrow \boxed{y > 0}$

$\downarrow$

$\sqrt{x} \neq 0$

$x \neq 0 \rightarrow f(g(x)) \rightarrow y \neq 0$.

$\therefore \boxed{x > 0}$

$D_{fg} = \{x : x \in \mathbb{R}, x > 0\}$

$R_{fg} = \{y : y \in \mathbb{R}, y > 0\}$.

Q18 $f(x) = x + 3$

$g(x) = \sqrt{x}$

$f(g(x)) = \sqrt{x} + 3$

$x \geq 0 \rightarrow \boxed{g(x)} \rightarrow y \geq 0$

$\downarrow$

$x \in \mathbb{R} \rightarrow \boxed{f(x)} \rightarrow y \in \mathbb{R}$

$\downarrow$

$\boxed{x \geq 0} \rightarrow \boxed{f(g(x))} \rightarrow \boxed{y \geq 3}$

$\therefore$ $f(g(x))$ is a function for the natural domain of $g(x)$ because it is one-to-one $\forall x, x \geq 0$.

but $g(f(x)) = \sqrt{x+3}$

$x \in \mathbb{R} \rightarrow \boxed{f(x)} \rightarrow y \in \mathbb{R}$

$\downarrow$

$x \geq 0 \rightarrow \boxed{g(x)} \rightarrow \boxed{y \geq 0}$

$\downarrow$

$x + 3 \geq 0$

$\boxed{x \geq -3} \rightarrow g(f(x)) \rightarrow \boxed{y \geq 0}$

is not a function over $x \in \mathbb{R}$, because when $x < -3$, it does not exist in $\mathbb{R}$.

Q19. $f(x) = x + 3$

$g(x) = \dfrac{1}{x-5}$

$f(g(x)) = \dfrac{1}{x-5} + 3$

$x \neq 5 \rightarrow \boxed{g(x)} \rightarrow y \neq 0.$

$\downarrow$

$x \in \mathbb{R} \rightarrow \boxed{f(x)} \rightarrow y \in \mathbb{R}.$

$\downarrow$

$\boxed{x \neq 5} \rightarrow \boxed{f(g(x))} \rightarrow \boxed{y \neq 3}$

$\therefore$ $f(g(x))$ is a function for the natural domain of $g(x)$ as it is consistent and $f(g(x))$ is one-to-one.

but $g(f(x)) = \dfrac{1}{x+3-5}$

$= \dfrac{1}{x-2}$

$x \in \mathbb{R} \rightarrow \boxed{f(x)} \rightarrow y \in \mathbb{R}$

$\downarrow$

$x \neq 5 \rightarrow \boxed{g(x)} \rightarrow y \neq 0$

$\downarrow$

$x + 3 \neq 5$

$\boxed{x \neq 2} \rightarrow g(f(x)) \rightarrow \boxed{y \neq 0}$

is not a function over $x \in \mathbb{R}$, because $f(x) = 5$, and. hence $g(x)$ does not exist.

Q20. $f(x) = x^2 - 9$

$g(x) = \dfrac{1}{x}$

a) $g(f(x)) = \dfrac{1}{x^2-9}$

$x \in \mathbb{R} \rightarrow f(x) \rightarrow y \geq -9$

$\downarrow$

$x \neq 0 \rightarrow g(x) \rightarrow y \neq 0$

$\downarrow$

$x^2 - 9 \neq 0$

$x \neq \pm 3 \rightarrow g(f(x)) \rightarrow$

Given that $x^2 - 9$ is at min at $x = 0$, then local max of $\dfrac{1}{x^2-9}$ at $x = 0$

$\Rightarrow$ local max of $-\dfrac{1}{9}$.

As $f(x) \rightarrow \infty$, $\dfrac{1}{f(x)} \rightarrow 0$.

$\therefore$ $D_{gf} = \{x : x \in \mathbb{R}, x \neq \pm 3\}$

$R_{gf} = \{y : y \in \mathbb{R}, y \leq -\frac{1}{9} \cup y > 0\}$.

b)

$f(g(x)) = \frac{1}{x^2} - 9$

$$x \neq 0 \rightarrow \boxed{g(x)} \rightarrow y \neq 0$$
$$\downarrow$$
$$x \in \mathbb{R} \rightarrow \boxed{f(x)} \rightarrow y \geq -9$$
$$\downarrow$$
$$\boxed{x \neq 0} \rightarrow f(g(x)) \rightarrow \boxed{y > -9}$$

$\therefore D_{fg} = \{x : x \in \mathbb{R}, x \neq 0\}$

$R_{fg} = \{y : y \in \mathbb{R}, y > -9\}$

EXERCISE 3B

Q1.

a) $f(x) = x$, $D_f = \{x : x \in \mathbb{R}\}$, $R_f = \{y : y \in \mathbb{R}\}$

$f^{-1}(x) = x$, $D_{f^{-1}} = \{x : x \in \mathbb{R}\}$, $R_{f^{-1}} = \{y : y \in \mathbb{R}\}$.

$\therefore$ Inverse on natural domain, as one-to-one

b) $f(x) = 2x + 3$, $D_f = \{x : x \in \mathbb{R}\}$, $R_f = \{y : y \in \mathbb{R}\}$

Let $y = 2x + 3$

$x \Leftrightarrow y$, $x = 2y + 3$

$\frac{x-3}{2} = y$

$\therefore f^{-1}(x) = \frac{x-3}{2}$, $D_{f^{-1}} = \{x : x \in \mathbb{R}\}$, $R_{f^{-1}} = \{y : y \in \mathbb{R}\}$

$\therefore$ Inverse on natural domain, as one-to-one

c) $f(x) = 5x - 3$, $D_f = \{x : x \in \mathbb{R}\}$, $R_f = \{y : y \in \mathbb{R}\}$

let $y = 5x - 3$

$x \Leftrightarrow y$, $x = 5y - 3$

$\frac{x+3}{5} = y$

$\therefore f^{-1}(x) = \frac{x+3}{5}$, $D_{f^{-1}} = \{x : x \in \mathbb{R}\}$, $R_{f^{-1}} = \{y : y \in \mathbb{R}\}$

$\therefore$ Inverse on natural domain, as one-to-one.

d) $f(x) = x^2$, $D_f = \{x : x \in \mathbb{R}\}$, $R_f = \{y : y \in \mathbb{R}, y \geq 0\}$

let $y = x^2$

$x \Leftrightarrow y$, $x = y^2$

$y = \pm\sqrt{x}$

$\therefore f^{-1}(x) = \pm\sqrt{x}$ $D_{f^{-1}} = \{x : x \in \mathbb{R}, x \geq 0\}$, $R_{f^{-1}} = \{y : y \in \mathbb{R}\}$

But not a function, as one-to-many over natural domain.

e) $f(x) = (2x-1)^2$, $D_f = \{x : x \in \mathbb{R}\}$, $R_f = \{y : y \in \mathbb{R}, y \geq 0\}$

Let $y = (2x-1)^2$

$x \Leftrightarrow y$, $x = (2y-1)^2$

$\pm\sqrt{x} = 2y - 1$

$\pm\sqrt{x} + 1 = 2y$

$y = \dfrac{\pm\sqrt{x}+1}{2}$

$\therefore f^{-1}(x) = \dfrac{\pm\sqrt{x}+1}{2}$, $D_{f^{-1}} = \{x : x \in \mathbb{R}, x \geq 0\}$, $R_{f^{-1}} = \{y : y \in \mathbb{R}\}$

But not a function as one-to-many over natural domain.

f) $f(x) = x^2 + 4$, $D_f = \{x : x \in \mathbb{R}\}$, $R_f = \{y : y \in \mathbb{R}, y \geq 4\}$

Let $y = x^2 + 4$

$x \Leftrightarrow y$, $x = y^2 + 4$

$x - 4 = y^2$

$y = \pm\sqrt{x-4}$

$\therefore f^{-1}(x) = \pm\sqrt{x-4}$, $D_{f^{-1}} = \{x : x \in \mathbb{R}, x \geq 4\}$, $R_{f^{-1}} = \{y : y \in \mathbb{R}\}$

But not a function as one-to-many over natural domain.

g) $f(x) = \frac{1}{x}$, $D_f = \{x : x \in \mathbb{R}, x \neq 0\}$, $R_f = \{y : y \in \mathbb{R}, y \neq 0\}$

Let $y = \frac{1}{x}$

$x \Leftrightarrow y$, $x = \frac{1}{y}$

$y = \frac{1}{x}$

$\therefore f^{-1}(x) = \frac{1}{x}$, $D_{f^{-1}} = \{x : x \in \mathbb{R}, x \neq 0\}$, $R_{f^{-1}} = \{y : y \in \mathbb{R}, y \neq 0\}$

$\therefore$ Inverse on natural domain, as one-to-one.

h) $f(x) = \dfrac{1}{x-3}$, $D_f = \{x : x \in \mathbb{R}, x \neq 3\}$, $R_f = \{y : y \in \mathbb{R}, y \neq 0\}$

Let $y = \dfrac{1}{x-3}$

$x \Leftrightarrow y$, $x = \dfrac{1}{y-3}$

$y - 3 = \frac{1}{x}$

$y = \frac{1}{x} + 3$

$\therefore f^{-1}(x) = \frac{1}{x} + 3$, $D_{f^{-1}} = \{x : x \in \mathbb{R}, x \neq 0\}$, $R_{f^{-1}} = \{y : y \in \mathbb{R}, y \neq 3\}$

$\therefore$ Inverse on natural domain, as one-to-one.

i) $f(x) = \frac{1}{x^2}$, $D_f = \{x : x \in \mathbb{R}, x \neq 0\}$, $R_f = \{y : y \in \mathbb{R}, y > 0\}$

Let $y = \frac{1}{x^2}$

$x \Leftrightarrow y$, $x = \frac{1}{y^2}$

$y^2 = \frac{1}{x}$

$y = \pm \frac{1}{\sqrt{x}}$

$\therefore f^{-1}(x) = \pm \frac{1}{\sqrt{x}}$, $D_{f^{-1}} = \{x : x \in \mathbb{R}, x > 0\}$, $R_{f^{-1}} = \{y : y \in \mathbb{R}, y \neq 0\}$

But not a function, as one-to-many over natural domain.

Q2 $f(x) = x - 2$

Let $y = x - 2$

$x \Leftrightarrow y$, $x = y - 2$

$y = x + 2$

$\therefore f^{-1}(x) = x + 2$, $D_{f^{-1}} = \{x : x \in \mathbb{R}\}$, $R_{f^{-1}} = \{y : y \in \mathbb{R}\}$.

Q3. $f(x) = 2x - 5$

Let $y = 2x - 5$

$x \Leftrightarrow y$, $x = 2y - 5$

$x + 5 = 2y$

$y = \frac{x+5}{2}$

$\therefore f^{-1}(x) = \frac{x+5}{2}$, $D_{f^{-1}} = \{x : x \in \mathbb{R}\}$, $R_{f^{-1}} = \{y : y \in \mathbb{R}\}$

Q4. $f(x) = 5x + 2$

Let $y = 5x + 2$

$x \Leftrightarrow y$, $x = 5y + 2$

$x - 2 = 5y$

$y = \frac{x-2}{5}$

$\therefore f^{-1}(x) = \frac{x-2}{5}$, $D_{f^{-1}} = \{x : x \in \mathbb{R}\}$, $R_{f^{-1}} = \{y : y \in \mathbb{R}\}$

Q5. $f(x) = \frac{1}{x-4}$

Let $y = \frac{1}{x-4}$

$x \Leftrightarrow y$, $x = \frac{1}{y-4}$

$y = \frac{1}{x} + 4$, $D_{f^{-1}} = \{x : x \in \mathbb{R}, x \neq 0\}$, $R_{f^{-1}} = \{y : y \in \mathbb{R}, y \neq 4\}$

Q6. $f(x) = \frac{1}{x+3}$

Let $y = \frac{1}{x+3}$,

$x \Leftrightarrow y$, $x = \frac{1}{y+3}$

$y+3 = \frac{1}{x}$

$y = \frac{1}{x} - 3$

$\therefore f^{-1}(x) = \frac{1}{x} - 3$, $D_{f^{-1}} = \{x : x \in \mathbb{R}, x \neq 0\}$, $R_{f^{-1}} = \{y : y \in \mathbb{R}, y \neq -3\}$.

Q7. $f(x) = \frac{1}{2x-5}$

Let $y = \frac{1}{2x-5}$,

$x \Leftrightarrow y$, $x = \frac{1}{2y-5}$

$2y-5 = \frac{1}{x}$

$2y = \frac{1}{x} + 5$

$y = \frac{2}{x} + 10$

$\therefore f^{-1}(x) = \frac{2}{x} + 10$, $D_{f^{-1}} = \{x : x \in \mathbb{R}, x \neq 0\}$, $R_{f^{-1}} = \{y : y \in \mathbb{R}, y \neq 10\}$

Q8. $f(x) = 1 + \frac{1}{2+x}$

Let $y = 1 + \frac{1}{2+x}$

$x \Leftrightarrow y$, $x = 1 + \frac{1}{2+y}$

$x - 1 = \frac{1}{2+y}$

$2 + y = \frac{1}{x-1}$

$y = \frac{1}{x-1} - 2$

$\therefore f^{-1}(x) = \frac{1}{x-1} - 2$, $D_{f^{-1}} = \{x : x \in \mathbb{R}, x \neq 1\}$, $R_{f^{-1}} = \{y : y \in \mathbb{R}, y \neq -2\}$.

Q9. $f(x) = 3 - \frac{1}{x-1}$

Let $y = 3 - \frac{1}{x-1}$

$x \Leftrightarrow y$, $x = 3 - \frac{1}{y-1}$

$x - 3 = -\frac{1}{y-1}$

$y - 1 = \frac{1}{3-x}$

$y = \frac{1}{3-x} + 1$,

$\therefore f^{-1}(x) = \frac{1}{3-x} + 1$, $D_{f^{-1}} = \{x : x \in \mathbb{R}, x \neq 3\}$ $R_{f^{-1}} = \{y : y \in \mathbb{R}, y \neq 1\}$

Q10. $f(x) = 4 + \frac{2}{2x-1}$

Let $y = 4 + \frac{2}{2x-1}$

$x \Leftrightarrow y$, $x = 4 + \frac{2}{2y-1}$

$x - 4 = \frac{2}{2y-1}$

$2y - 1 = \frac{2}{x-4}$

$2y = \frac{2}{x-4} + 1$

$y = \frac{1}{x-4} + \frac{1}{2}$

$\therefore f^{-1}(x) = \frac{1}{x-4} + \frac{1}{2}$

$D_{f^{-1}} = \{x : x \in \mathbb{R}, x \neq 4\}$

$R_{f^{-1}} = \{y : y \in \mathbb{R}, y \neq \frac{1}{2}\}$.

Q11. $f(x) = \sqrt{x}$

Let $y = \sqrt{x}$

$x \Leftrightarrow y$, $x = \sqrt{y}$

$y = x^2$

$\therefore f^{-1}(x) = x^2$,

but $D_{f^{-1}} = \{x : x \in \mathbb{R}, x \geq 0\}$

$R_{f^{-1}} = \{y : y \in \mathbb{R}, y \geq 0\}$.

Q12. $f(x) = \sqrt{x+1}$

Let $y = \sqrt{x+1}$

$x \Leftrightarrow y$, $x = \sqrt{y+1}$

$y + 1 = x^2$

$y = x^2 - 1$

$\therefore f^{-1}(x) = x^2 - 1$

but $D_{f^{-1}} = \{x : x \in \mathbb{R}, x \geq 0\}$

$R_{f^{-1}} = \{y : y \in \mathbb{R}, y \geq -1\}$.

Q13. $f(x) = \sqrt{2x-3}$

Let $y = \sqrt{2x-3}$

$x \Leftrightarrow y$, $x = \sqrt{2y-3}$

$x^2 = 2y - 3$

$2y = x^2 + 3$

$y = \frac{x^2+3}{2}$

$\therefore f^{-1}(x) = \frac{x^2+3}{2}$

$D_{f^{-1}} = \{x : x \in \mathbb{R}, x \geq 0\}$.

$R_{f^{-1}} = \{y : y \in \mathbb{R}, y \geq \frac{3}{2}\}$

$f(x) = 2x + 5$, $g(x) = 3x + 1$,

and $h(x) = 1 + \frac{2}{x}$

Q14. $f^{-1}(x)$.

Let $y = 2x + 5$, $x \Leftrightarrow y$.

$x = 2y + 5$

$2y = x - 5$

$y = \frac{x-5}{2}$

$\therefore f^{-1}(x) = \frac{x-5}{2}$.

Q15. $g^{-1}(x)$

Let $y = 3x + 1$, $x \Leftrightarrow y$

$x = 3y + 1$

$3y = x - 1$

$y = \frac{x-1}{3}$

$\therefore g^{-1}(x) = \frac{x-1}{3}$.

Q16. $h^{-1}(x)$

Let $y = 1 + \frac{2}{x}$, $x \Leftrightarrow y$

$x = 1 + \frac{2}{y}$

$x - 1 = \frac{2}{y}$

$y = \frac{2}{x-1}$

$\therefore h^{-1}(x) = \frac{2}{x-1}$.

Q17. $f \circ f^{-1}(x)$

$= 2\left(\frac{x-5}{2}\right) + 5$

$= x - 5 + 5$

$= \underline{\underline{x}}$

Q18. $f^{-1} \circ f(x)$

$= \frac{(2x+5) - 5}{2}$

$= \frac{2x}{2}$

$= \underline{\underline{x}}$

Q19. $f \circ h^{-1}(x)$

$= 2\left(\frac{2}{x-1}\right) + 5$

$= \frac{4}{x-1} + 5$

Q20. $(f \circ g)^{-1}(x)$

$f \circ g(x) = 2(3x+1)+5$

$= 6x+2+5$

$= 6x+7$

$\therefore (f \circ g)^{-1}(x)$

let $y = 6x+7$, $x \Leftrightarrow y$

$x = 6y+7$

$y = \frac{x-7}{6}$

$\therefore (f \circ g)^{-1}(x) = \frac{x-7}{6}$

Q21. $g^{-1} \circ f^{-1}(x)$

$= \frac{\left(\frac{x-5}{2}\right) - 1}{3}$

$= \frac{\left(\frac{x-5-2}{2}\right)}{3}$

$= \frac{x-7}{6}$

⊛ NOTE: $(f \circ g)^{-1}(x) = g^{-1} \circ f^{-1}(x)$.

Q22. $f \circ g^{-1}(x)$

$= 2\left(\frac{x-1}{3}\right) + 5$

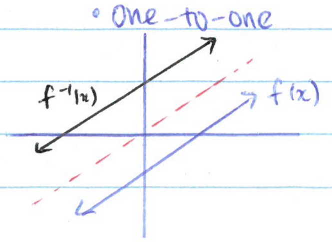
$= \frac{2x-2}{3} + 5$

$= \frac{2x-2+15}{3}$

$= \frac{2x+13}{3}$

Q23 a) • Linear function

• One-to-one



b) Parabolic relation

one-to-many

c) linear function

many-to-one

d) circular relation

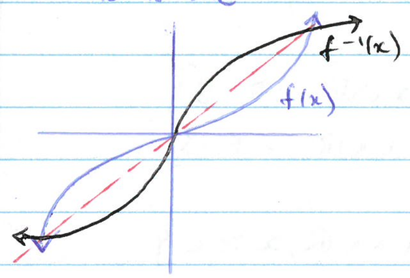
many-to-many.

e) cubic relation

one-to-many

f) odd degree function

one-to-one



Q24

$f(x) = x^2+3$

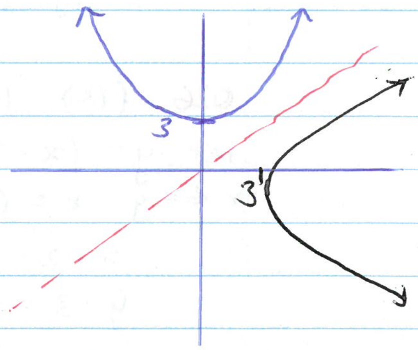
let $y = x^2+3$

$x \Leftrightarrow y$, $x = y^2+3$

$y^2 = x-3$

$y = \pm\sqrt{x-3}$

$\therefore f^{-1}(x) = \pm\sqrt{x-3}$



For $f^{-1}(x)$ to exist as a function, either:

$D_f = \{x : x \in \mathbb{R}, x \geq 0\}$ or $[+\sqrt{x-3}]$

$= \{x : x \in \mathbb{R}, x \leq 0\}$. $[-\sqrt{x-3}]$.

such that

$D_{f^{-1}} = \{x : x \in \mathbb{R}, x \geq 3\}$

$R_{f^{-1}} = \{y : y \in \mathbb{R}, y \geq 0\}$, or $[+\sqrt{x-3}]$

$\{y : y \in \mathbb{R}, y \leq 0\}$ $[-\sqrt{x-3}]$.

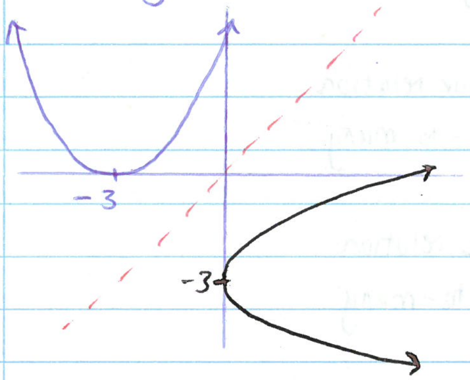
Q25. $f(x) = (x+3)^2$

Let $y = (x+3)^2$

$x \Leftrightarrow y, \quad x = (y+3)^2$

$\pm\sqrt{x} = y+3$

$y = \pm\sqrt{x} - 3 \Rightarrow f^{-1}(x)$



For $f^{-1}(x)$ to exist as a function,

either:

$D_f = \{x : x \in \mathbb{R}, x \geq -3\}$

or $= \{x : x \in \mathbb{R}, x \leq -3\}$.

such that

$D_{f^{-1}} = \{x : x \in \mathbb{R}, x \geq 0\}$

$R_{f^{-1}} = \{y : y \in \mathbb{R}, x \geq -3\}$, or

$\{y : y \in \mathbb{R}, y \leq -3\}$.

Q26. $f(x) = (x-3)^2 + 2$

Let $y = (x-3)^2 + 2$

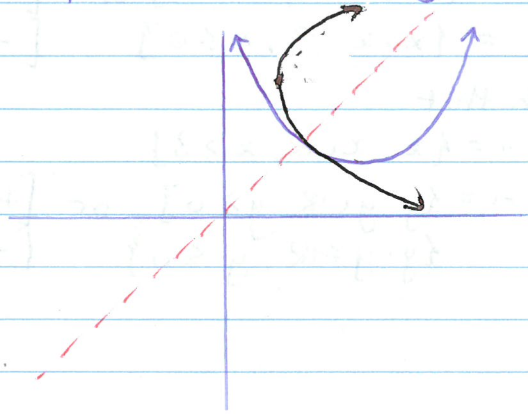
$x \Leftrightarrow y, \quad x = (y-3)^2 + 2$

$x - 2 = (y-3)^2$

$y - 3 = \pm\sqrt{x-2}$

$y = \pm\sqrt{x-2} + 3$

$\therefore f^{-1}(x) = \pm\sqrt{x-2} + 3$



For $f^{-1}(x)$ to exist as a function,

either:

$D_f = \{x : x \in \mathbb{R}, x \geq 3\}$ or

$\{x : x \in \mathbb{R}, x \leq 3\}$.

such that

$D_{f^{-1}} = \{x : x \in \mathbb{R}, x \geq 2\}$

$R_{f^{-1}} = \{y : y \in \mathbb{R}, y \geq 3\}$ or

$\{y : y \in \mathbb{R}, y \leq 3\}$.

Q27. $f(x) = \sqrt{4-x^2}$

let $y = \sqrt{4-x^2}$

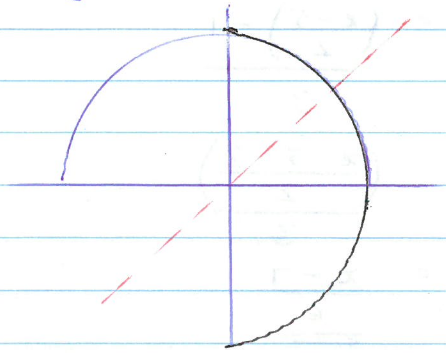
$x \Leftrightarrow y, \quad x = \sqrt{4-y^2}$

$x^2 = 4 - y^2$

$-y^2 = x^2 - 4$

$y^2 = 4 - x^2$

$y = \pm\sqrt{4-x^2} \Rightarrow f^{-1}(x)$



For $f^{-1}(x)$ to exist as a function,

either:

$D_f = \{x : x \in \mathbb{R}, -2 \leq x \leq 0\}$ or

$\{x : x \in \mathbb{R}, 0 \leq x \leq 2\}$

such that

$D_{f^{-1}} = \{x : x \in \mathbb{R}, 0 \leq x \leq 2\}$

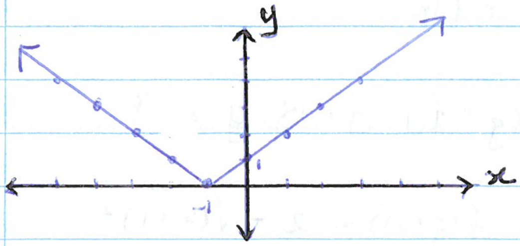
$R_{f^{-1}} = \{y : y \in \mathbb{R}, -2 \leq x \leq 0\}$ or

$\{y : y \in \mathbb{R}, 0 \leq x \leq 2\}$.

EXERCISE 3C

Q1. $y = |x+1|$

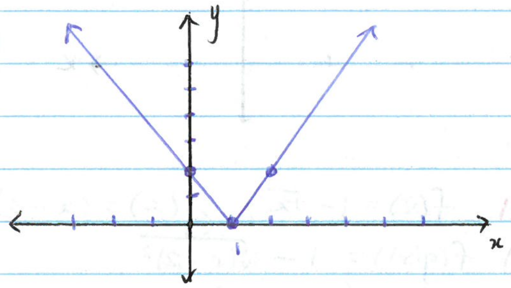
Fracture point : $x = -1$. $(-1, 0)$



Q2 $y = |2x - 2|$

Fracture point : $x = 1$, $(1, 0)$

y-intercept : 2.

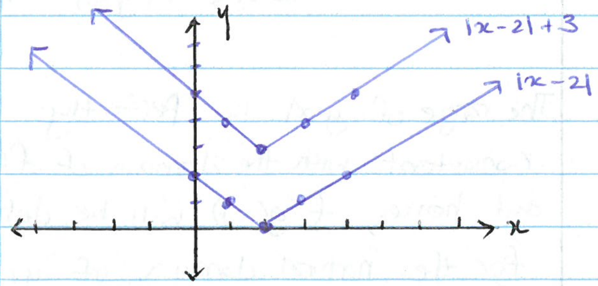


Q3 $y = |x-2|$

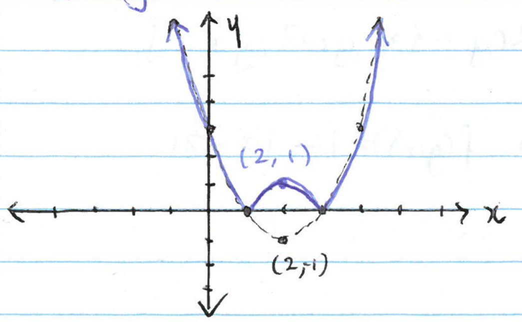
Fracture point : $x = 2$, $(2, 0)$

$y = |x-2| + 3$

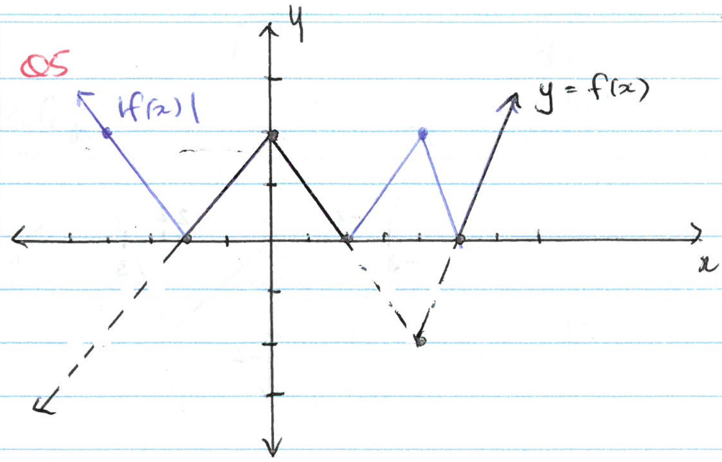
Fracture point : $x = 2$, $(2, 3)$



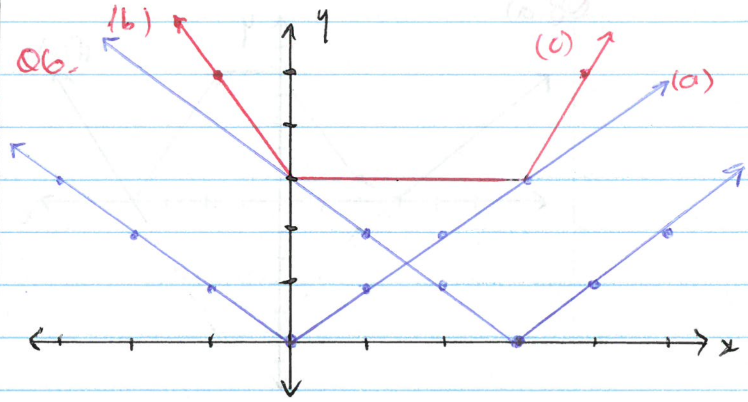
Q4. $y = |(x-2)^2 - 1|$



Q5



Q6.



a) $y = |x|$, fracture point at $x = 0$, $(0,0)$

b) $y = |x-3|$, fracture point at $x = 3$, $(3,0)$

c) $y = |x| + |x-3|$

For $x \leq 0$, $y = -x - (x-3)$

$= -x - x + 3$

$= -2x + 3$

For $0 < x < 3$, $y = x - (x-3)$

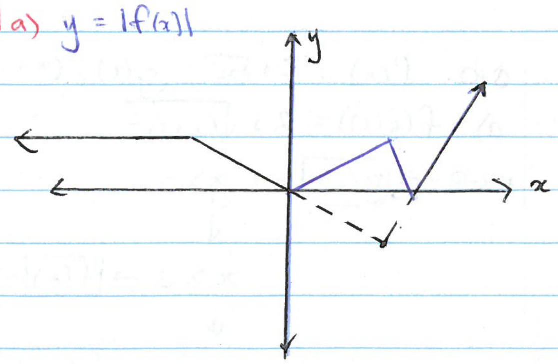
$= 3$

For $x \geq 3$, $y = x + x - 3$

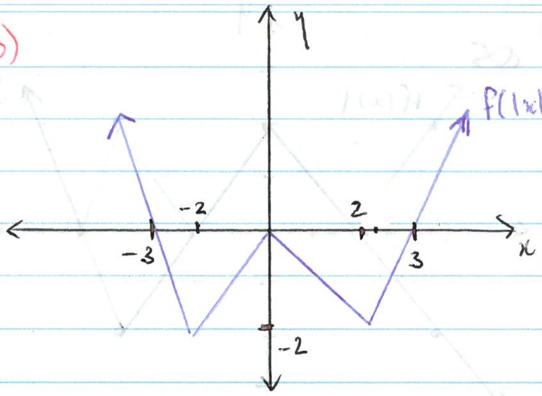
$= 2x - 3$

$$\therefore \quad y = \begin{cases} -2x+3, & x \leq 0 \\ 3, & 0 < x < 3 \\ 2x-3, & x \geq 3. \end{cases}$$

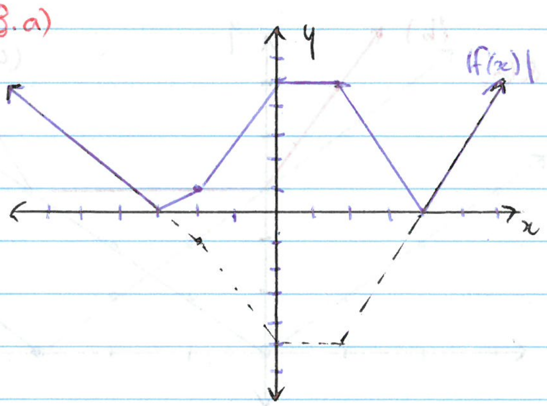
Q7 a) $y = |f(x)|$



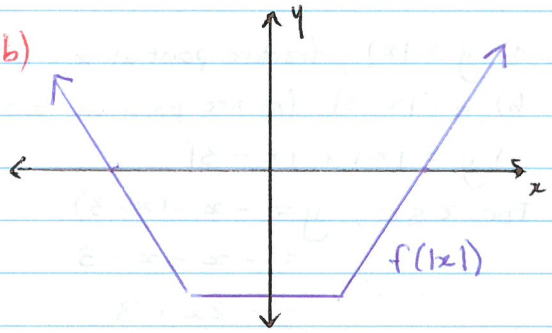
b)



Q8. a)



b)



Q9. All values of $g(x)$ for $x \geq 0$ are mirrored about the y-axis and replaces the values of $g(x)$ for $x < 0$.

i.e. $g(x) = g(|x|)$; $x \geq 0$.

but $g(|x|) = g(-x)$
for $x < 0$ for $x \geq 0$.

Q10. $f(x) = 2 + \sqrt{x}$, $g(x) = (x+1)^2$

a) $f(g(x)) = 2 + \sqrt{(x+1)^2}$

$x \in \mathbb{R} \rightarrow \boxed{g(x)} \rightarrow y \geq 0$.

$\downarrow$

$x \geq 0 \rightarrow \boxed{f(x)} \rightarrow y \geq 2$

$\downarrow$

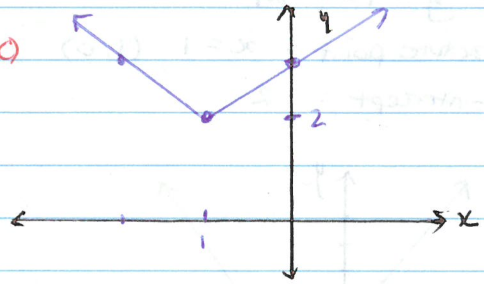
$x \in \mathbb{R} \rightarrow \boxed{f(g(x))} \rightarrow y \geq 2$

The range of $g(x)$ is perfectly consistent with the domain of $f(x)$ and hence, $f(g(x))$ will be defined for the natural domain of $g(x)$, which is $x \in \mathbb{R}$.

$R_{fg} = \{y : y \in \mathbb{R}, y \geq 2\}$.

b) $f(g(x)) = 2 + \sqrt{(x+1)^2}$
$= 2 + |x+1|$

c)



Q11. $f(x) = 1 - \sqrt{x}$, $g(x) = (x-2)^2$

a) $f(g(x)) = 1 - \sqrt{(x-2)^2}$

$x \in \mathbb{R} \rightarrow \boxed{g(x)} \rightarrow y \geq 0$

$\downarrow$

$x \geq 0 \rightarrow \boxed{f(x)} \rightarrow y \leq 1$

$\downarrow$

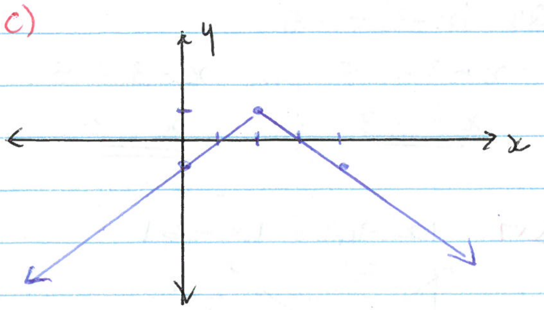
$x \in \mathbb{R} \rightarrow f(g(x)) \rightarrow y \leq 1$.

The range of $g(x)$ is perfectly consistent with the domain of $f(x)$ and hence, $f(g(x))$ will be defined for the natural domain of $g(x)$, which is $x \in \mathbb{R}$.

$R_{fg} = \{y : y \in \mathbb{R}, y \leq 1\}$.

b) $f(g(x)) = 1 - |x-2|$.

c)



Q12.

a) $|10-2x| = 4$

Graphically, $x=3$, $x=7$.

Algebraically,

$10-2x = 4$

$-2x = -6$

$\underline{\underline{x = 3}}$

$-(10-2x) = 4$

$10-2x = -4$

$-2x = -14$

$\underline{\underline{x = 7}}$

b) $|x-2| = 4$

Graphically, $x=-2$, $x=6$

Algebraically,

$x-2 = 4$

$\underline{\underline{x = 6}}$

$-(x-2) = 4$

$x-2 = -4$

$\underline{\underline{x = -2}}$

c) $|10-2x| = |x-2|$

Graphically, $x=4$, $x=8$

Algebraically,

$-(10-2x) = x-2$

$-10+2x = x-2$

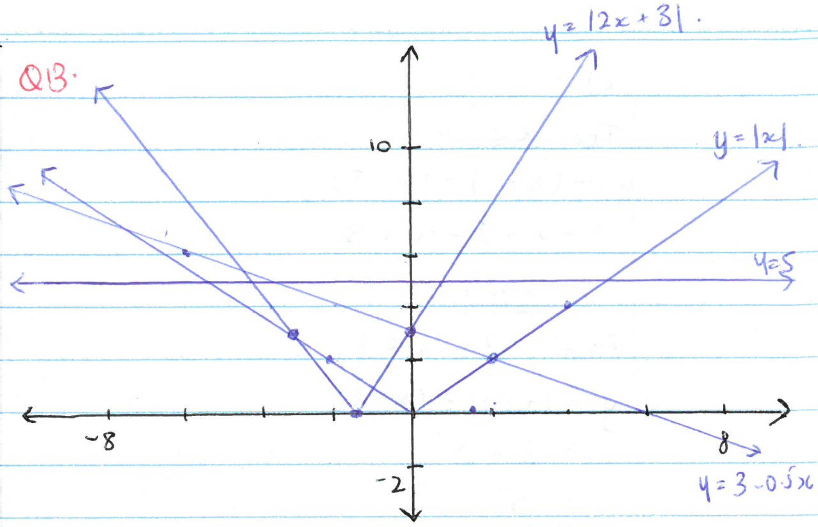
$\underline{\underline{x = 8}}$

$10-2x = x-2$

$-3x = -12$

$\underline{\underline{x = 4}}$

Q13.



a) $|2x+3| = 5$

$2x+3 = 5$

$2x = 2$

$\underline{\underline{x = 1}}$

$-(2x+3) = 5$

$2x+3 = -5$

$2x = -8$

$\underline{\underline{x = -4}}$

b) $3-0.5x = |x|$

$3-0.5x = x$, $3-0.5x = -x$

$3 = 1.5x$

$\underline{\underline{x = 2}}$

$3 = -0.5x$

$\underline{\underline{x = -6}}$

c) $3-0.5x = |2x+3|$.

$3-0.5x = 2x+3$

$-2.5x = 0$

$\underline{\underline{x = 0.}}$

$3-0.5x = -2x-3$

$1.5x = -6$

$\underline{\underline{x = -4}}$

d) $|x| = |2x+3|$

$-x = 2x+3$

$-3x = 3$

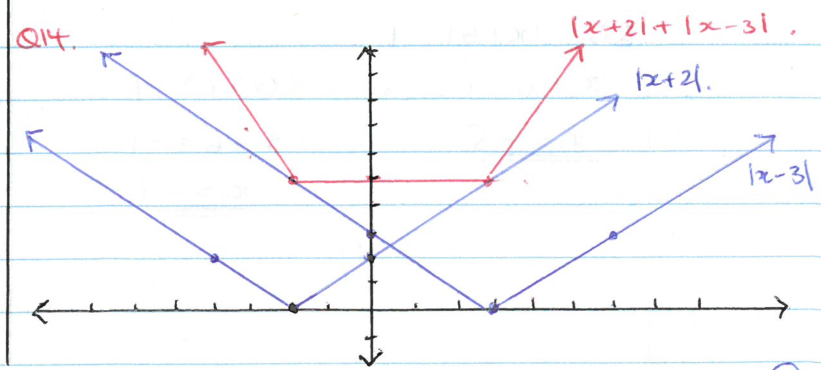
$\underline{\underline{x = -1}}$

$x = 2x+3$

$-x = 3$

$\underline{\underline{x = -3}}$

Q14.



c)

For $x \le -2$,

$y = -(x+2) - (x-3)$

$= -x - 2 - x + 3$

$= \underline{\underline{-2x+1}}$

For $-2 < x < 3$,

$y = (x+2) - (x-3)$

$= 2 + 3$

$= \underline{\underline{5}}$

For $x \ge 3$,

$y = (x+2) + (x-3)$

$= \underline{\underline{2x-1}}$

$$y = \begin{cases} -2x+1, & x \le -2 \\ 5, & -2 \le x < 3 \\ 2x-1, & x \ge 3. \end{cases}$$

d) $|x+2| + |x-3| \le 9$

[can deduce graphically, or...]

$-2x+1 \le 9$

$-2x \le 8$

$\boxed{x \ge -4}$

$5 \le 9$ $\quad \boxed{-2 \le x < 3}$

$2x - 1 \le 9$

$2x \le 10$

$\boxed{x \le 5}$

$\therefore |x+2| + |x-3| \le 9$

for $\underline{\underline{-4 \le x \le 5}}$.

Q15 $|x+6| = 1$

$x + 6 = 1$ or $-(x+6) = 1$

$\underline{\underline{x = -5}}$ $\quad x + 6 = -1$

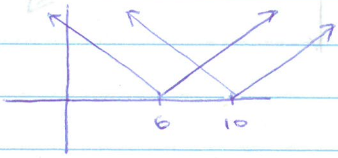
$\underline{\underline{x = -7}}$

Q16 $|x-3| = -5$

$x - 3 = -5$ or $x - 3 = 5$

$\underline{\underline{x = -2}}$ $\quad \underline{\underline{x = 8}}$

Q17. $|x-10| = |x-6|$



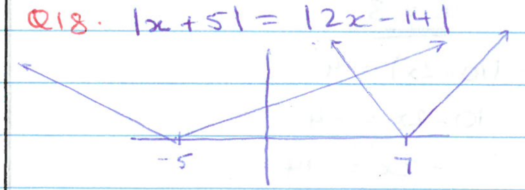
$\therefore -(x-10) = x - 6$

$-x + 10 = x - 6$

$-2x = -16$

$\underline{\underline{x = 8}}$

Q18. $|x+5| = |2x-14|$



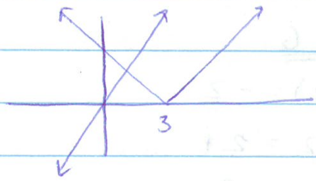
$x + 5 = -(2x-14)$ or $x + 5 = 2x - 14$

$x + 5 = -2x + 14$ $\quad x + 5 = 2x - 14$

$3x = 9$ $\quad -x = -19$

$\underline{\underline{x = 3}}$ $\quad \underline{\underline{x = 19}}$

Q19. $|x-3| = 2x$



$\therefore -(x-3) = 2x$

$-x + 3 = 2x$

$-3x = -3$

$\underline{\underline{x = 1}}$

Q20. $|x+5| + |x-1| = 7$.

$$y = \begin{cases} -2x-4, & x \le -5 \\ 6, & -5 < x < 1 \\ 2x+4, & x \ge 1. \end{cases}$$

$-2x - 4 = 7$ $\quad 2x + 4 = 7$

$-2x = 11$ $\quad 2x = 3$

$x = -\frac{11}{2}$ // $\quad x = \frac{3}{2}$ //

Q21

$|x+5| + |x-3| = 8$

$$y = \begin{cases} -2x-2, & x \le -5 \\ 8, & -5 < x < 3 \\ 2x+2, & x \ge 3 \end{cases}$$

$-2x-2=8$ $8=8$ $2x+2=8$

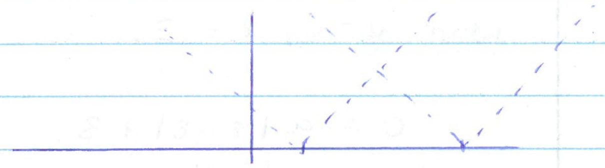
$-2x=10$ $\Downarrow$ $2x=6$

$x=-5$ $-5<x<3$ $x=3.$

$\therefore \ \underline{\underline{-5 \le x \le 3.}}$

Q22 $|x-8| = |2-x| - 6$

$|x-8| - |2-x| = -6$



Note: $|2-x|$ is the negative function already as $2-x = -(x-2)$.

For $x \le 2$,

$y = -(x-8) - (2-x)$,

$= -x+8-2+x$

$= 6$

For $2 \le x < 8$,

$y = -(x-8) + (2-x)$

$= -x+8+2-x$

$= -2x+10$

For $x \ge 8$,

$y = x-8 + (2-x)$,

$= x-8+2-x$

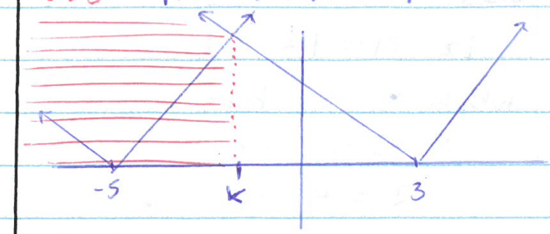
$= \underline{\underline{-6}}$

$\therefore |x-8| = |2-x| - 6$

$\Rightarrow |x-8| - |2-x| = -6$

$\underline{\underline{x \ge 8}}$

Q23. $|x-3| \ge |x+5|$



$-(x-3) = x+5$

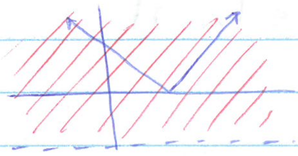
$-x+3 = x+5$

$-2x = 2$

$\underline{\underline{x=-1}}$

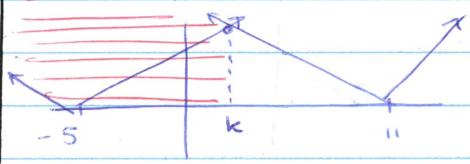
$\therefore \ \underline{\underline{x \le -1}}$

Q24. $|2x-5| \ge -5$



$\underline{\underline{x \in \mathbb{R}}}$.

Q25. $|x-11| \ge |x+5|$



$-(x-11) = x+5$

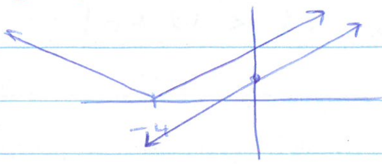
$-x+11 = x+5$

$-2x = -6$

$\underline{\underline{x=3}}$

$\therefore \ \underline{\underline{x \le 3}}$.

Q26. $|x+4| > x+2$



$\underline{\underline{x \in \mathbb{R}}}$

Q27. When $x=3$,

$2(3)+5 = 11$ $* \Rightarrow (>)$

$\therefore a = 11$

$\therefore -(2x+5) = 11$

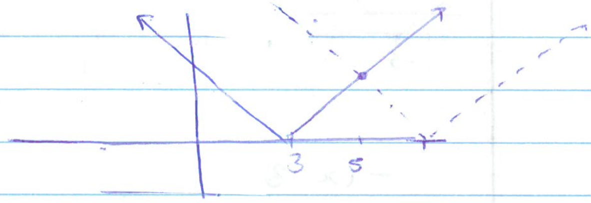
$2x = -16$

$x = -8$ //

Q28.

$|x-3| = |x-a|$

when $x=5$, but



$x-3 = -(x-a)$

$x-3 = -x+a$

$2x-3 = a$

$2(5)-3 = a$

$\underline{a = 7}$

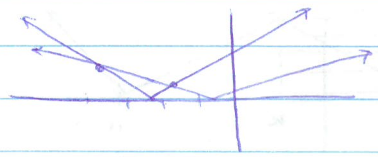
$\therefore\ * \Rightarrow \leq$

$|x-3| \leq |x-7|$.

Q29.

$|2x+5| = |x+a|$

when $x=-4$ and $x=-2$



$2x+5 = -(x+a)$, when $x=-2$

$2x+5 = -x-a$

$3x+5 = -a$

$3(-2)+5 = -a$

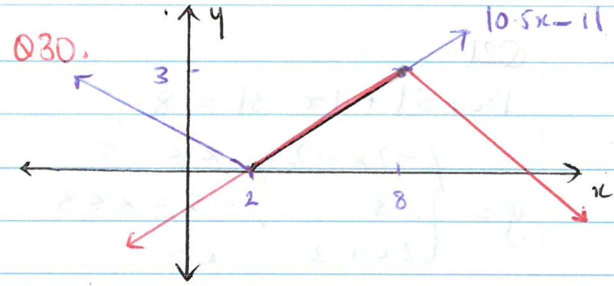
$-1 = -a$

$a = 1$

$* \Rightarrow <$

$\therefore |2x+5| < |x+1|$

Q30.



when $x=8$, $|0.5(8)-1| = \underline{3}$.

$\therefore \boxed{c=3}$

and $\boxed{b=8}$

$\therefore y = a|x-8|+3$.

When $y=0$, $x=2$.

$0 = a|2-8|+3$

$-3 = a|-6|$

$-3 = 6a$

$\boxed{a = -\frac{1}{2}}$

EXERCISE 3D

Q1. $y = \frac{2}{x}$

$\therefore x = 0$

Q2. $y = \frac{5}{x-1}$

$\therefore x = 1$

Q3. $y = \frac{5}{(x-3)(2x-1)}$

$\therefore x = 3,\ x = \frac{1}{2}$.

Q4. $y = \frac{x+3}{x-3}$

$\therefore x = 3$.

Q5. $y = \frac{3}{x}$

$\therefore y \neq 0$.

Q6. $y = 2 + \frac{3}{x}$

$\therefore y \neq 2$.

Q7. $y = \frac{1}{x+1}$

$\therefore y \neq 0$.

Q8 $y = \frac{x-1}{x+1}$

$= \frac{1(x+1)-2}{x+1}$

$= 1 - \frac{2}{x+1}$

$\therefore y \neq 1$.

Q9. $y = \frac{1}{x-5}$

As $x \to \infty$, $y \to 0^+$

As $x \to -\infty$, $y \to 0^-$

Q10. $y = \frac{x+2}{x-2}$

$= \frac{1(x-2)+4}{x-2}$

$= 1 + \frac{4}{x-2}$

$\therefore$ As $x \to \infty$, $y \to 1^+$

As $x \to -\infty$, $y \to 1^-$

Q11. $y = \frac{5x^2+7x-3}{x^2+6}$

$= \frac{5(x^2+6)+7x-33}{(x^2+6)}$

$= 5 + \frac{7x-33}{x^2+6}$

$\therefore$ As $x \to \infty$, $y \to 5^+$

As $x \to -\infty$, $y \to 5^-$.

or using the limit law:

$\lim_{x \to \infty} \frac{f(x)}{g(x)} = 5$

Q12. $y = \frac{3x(x+2)}{x^2+1}$

$= \frac{3x^2+6x}{x^2+1}$

$= \frac{3(x^2+1)+6x-3}{x^2+1}$

$= 3 + \frac{6x-3}{x^2+1}$

$\therefore$ As $x \to \infty$, $y \to 3^+$

As $x \to -\infty$, $y \to 3^-$

Q13. $y = \frac{1}{x-3}$

As $x \to 3^+$, $y \to \infty$

As $x \to 3^-$, $y \to -\infty$

Q14. $y = \frac{1}{1-x}$

As $x \to 1^+$, $y \to -\infty$

As $x \to 1^-$, $y \to \infty$

Q15 $y = \frac{x^5+1}{x^2}$

$= x^3 + \frac{1}{x^2}$

As $x \to 0^+$, $y \to \infty$

As $x \to 0^-$, $y \to \infty$

Q16 a) $x \neq 3$.

$R = \{y : y \in \mathbb{R}, y > 0\}$

$\therefore y = \frac{1}{(x-3)^2}$.

b) $x \neq \pm 3$

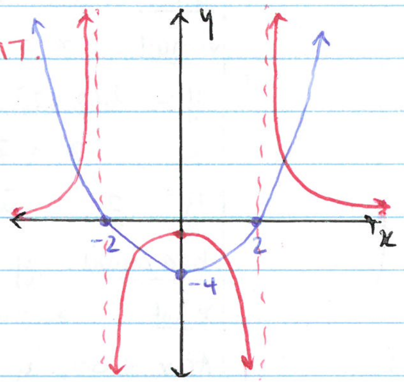
when $x = 0$, $y = -\frac{1}{9}$

$\therefore y = \frac{1}{(x+3)(x-3)}$

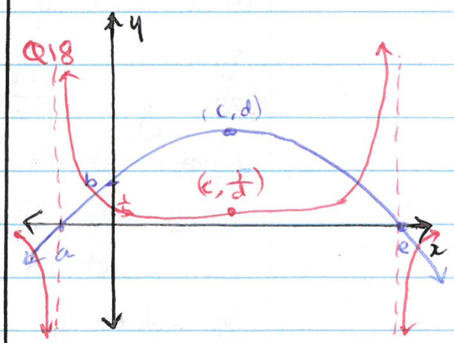
c) $x \neq 3$

$\therefore y = \frac{1}{x-3}$.

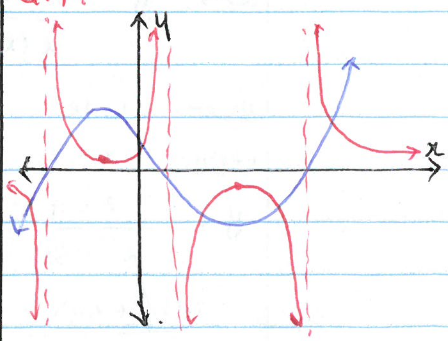
Q17.



Q18



Q19.



For questions 20 - 31, see Sadler for sketches. Function features are shown below.

Q20

$y = \frac{x+3}{x-1}$

y int: $(0, -3)$

vertical: $x \neq 1$

$y = \frac{1(x-1)+4}{x-1}$

$y = 1 + \frac{4}{x-1}$

horizontal: $y \neq 1$

x int: $(-3, 0)$

As $x \to \infty$, $y \to 0^+$

As $x \to -\infty$, $y \to 0^-$

Q21. $y = \frac{2x-4}{x+2}$

y int: $(0, -2)$

vertical: $x \neq -2$

$y = \frac{2(x+2) - 8}{x+2}$

$y = 2 - \frac{8}{x+2}$

horizontal: $y \neq 2$

x int: $(2, 0)$

As $x \to \infty$, $y \to 0^-$

As $x \to \infty$, $y \to 0^+$

Q22 $y = \frac{2(x-4)}{(x-4)}$

y-int: $(0, 2)$

vertical: point of discontinuity at $(4, 2)$

$y = 2.$

Q23. $y = \frac{x^2-9}{x(x+3)}$

y-int: none

vertical: $x \neq 0$, $x \neq -3$

$y = \frac{x^2-9}{x^2+3x}$

$= \frac{1(x^2+3x) - 3x - 9}{x^2+3x}$

$= 1 - \frac{3(x+3)}{x(x+3)}$

$y = 1 - \frac{3}{x}$

point of discontinuity at $x = -3$.

$(-3, 2)$

horizontal: $y \neq 1$

As $x \to \infty$, $y \to 1^-$

As $x \to -\infty$, $y \to 1^+$

Q24. $y = \frac{36(2-x)}{x(x+6)}$

As $x \to \infty$, $y \to 0^-$

As $x \to -\infty$, $y \to 0^+$

y-int: none

vertical: $x \neq 0$, $x \neq -6$.

$y = \frac{72 - 36x}{x^2+6x}$ $\left[\to \lim_{x \to \infty} \frac{-36}{x} \right]$.

x-int: $0 = 72 - 36x$

$x = 2$

$\therefore (2, 0)$.

extrema:

$\frac{dy}{dx} = \frac{x^2+6x(-36) - (2x+6)(72-36x)}{(x^2+6x)^2}$

$= \frac{-36x^2 - 216x - [144x - 72x^2 + 432 - 216x]}{(x^2+6x)^2}$

$0 = 36x^2 - 144x - 432$

$0 = x^2 - 4x - 12$

$0 = (x-6)(x+2)$

$\therefore x = 6$ and $x = -2$

$y = \frac{36(-4)}{6(12)}$ $\quad y = \frac{36(4)}{-2(4)}$

$= -2$ $\quad = -18$

$(6, -2)$ $\quad \therefore (-2, -18)$

Q25. $y = \frac{1-x}{x(x+3)}$

y-int: none

vertical: $x \neq 0$, $x \neq -3$

$y = \frac{1-x}{x^2+3x}$

x-int: $(1, 0)$

extrema:

$$y = \frac{1-x}{x^2+3x}$$

$$\frac{dy}{dx} = \frac{(x^2+3x)(-1) - (1-x)(2x+3)}{(x^2+3x)^2}$$

$$0 = -x^2-3x - [2x+3-2x^2-3x]$$

$$0 = -x^2-3x - [-2x^2-x+3]$$

$$0 = -x^2-3x+2x^2+x-3$$

$$0 = x^2-2x-3$$

$$0 = (x-3)(x+1)$$

$x = 3$ or $x = -1$

$y = \frac{1-3}{3(6)} = -\frac{1}{9}$ $\qquad y = \frac{2}{-1(2)} = -1$

$\therefore (3, -\frac{1}{9})$ $\qquad \therefore (-1,-1)$

$$\left[\lim_{x\to\infty} \frac{-1}{x}\right]$$

As $x \to \infty$, $y \to 0^-$

As $x \to -\infty$, $y \to 0^+$

Q26. $y = \frac{x}{x^2-1}$

$= \frac{x}{(x+1)(x-1)}$

y-int: $(0,0)$

vertical: $x \neq \pm 1$

x-int: $(0,0)$

extrema: $\frac{dy}{dx} = \frac{(x^2-1)(1)-(2x)(x)}{x^2-1}$

$0 = x^2-1-2x^2$

$0 = -x^2-1$

$0 = x^2+1$ (none)

inflection: $\frac{dy}{dx} = \frac{-x^2-1}{x^2-1}$

$= \frac{-1(x^2-1)-2}{x^2-1}$

$= -1 - \frac{2}{x^2-1}$

$\frac{d^2y}{dx^2} = \frac{4x}{(x^2-1)}$

$\therefore$ inflection at $(0,0)$

$$\left[\lim_{x\to\infty} \frac{1}{x}\right]$$

As $x \to \infty$, $y \to 0^+$

As $x \to -\infty$, $y \to 0^-$

Q27. $y = \frac{(x-4)(x-1)}{x-2}$

y-int: $y = \frac{(-4)(-1)}{(-2)}$

$= -2 \quad \therefore (0,-2)$

vertical: $x \neq 2$

$y = \frac{x^2-5x+4}{x-2}$

$= \frac{x(x-2)-3(x-2)-2}{x-2}$

$= x-3 - \frac{2}{x-2}$

oblique: $y \neq x-3$

x-int: $(4,0)$, $(1,0)$

As $x \to \infty$, $y \to (x-3)^-$

As $x \to -\infty$, $y \to (x-3)^+$

Q28. $y = \frac{x^2+3x}{x-1}$

y int: $(0,0)$, vertical: $x \neq 1$

$y = \frac{x(x-1)+4(x-1)+4}{x-1}$

$= x+4 + \frac{4}{x-1}$

oblique: $y \neq x+4$

x-int: $x^2+3x = 0$

$x(x+3) = 0$

$x = 0$, $x = -3$.

As $x \to \infty$, $y \to (x+4)^+$

As $x \to -\infty$, $y \to (x+4)^-$

extrema: $\frac{dy}{dx} = 1 - \frac{4}{(x-1)^2}$

$0 = 1 - \frac{4}{(x-1)^2}$

$(x-1)^2 = 4 \Rightarrow x = -1$, $x = 3$

$(-1,-1)$ $\qquad (3,9)$

Q29. $y = \dfrac{x^2-3x-4}{x^3-2x^2-3x}$

$= \dfrac{x^2-3x-4}{x(x^2-2x-3)}$

$= \dfrac{(x-4)(x+1)}{x(x-3)(x+1)}$

y int: none

vertical: $x \neq 0, x \neq 3, x \neq -1$

Point of discontinuity at $x=-1$.

$y = \dfrac{x-4}{x(x-3)} \Rightarrow y = \dfrac{-5}{-1(-4)} \Rightarrow (-1, -\frac{5}{4})$

extrema: $y = \dfrac{x-4}{x^2-3x}$

$\dfrac{dy}{dx} = \dfrac{(x^2-3x)(1)-(x-4)(2x-3)}{[x^2-3x]^2}$

$0 = x^2-3x-[2x^2-3x-8x+12]$

$0 = x^2-3x-2x^2+11x-12$

$0 = -x^2+8x-12$

$0 = x^2-8x+12$

$0 = (x-6)(x-2)$

$x=6$ $\qquad$ $x=2$

$y = \dfrac{2}{6(3)}$ $\qquad$ $y = \dfrac{-2}{2(-1)}$

$= \dfrac{1}{9}$ $\qquad$ $= 1$

$(6, \frac{1}{9})$ $\qquad$ $\therefore (2,1)$

As $x \to \infty, y \to 0^+$

As $x \to -\infty, y \to 0^-$

$\left[\lim_{x\to\infty} \dfrac{1}{x}\right]$

x int: $x=4$, $(4,0)$

Q30. $y = \dfrac{3}{x^3-3x^2+3x}$

$= \dfrac{3}{x(x^2-3x+3)}$

y int: none

vertical: $x \neq 0$

[no real solns to x^2-3x+3]

extrema:

$\dfrac{dy}{dx} = \dfrac{-3(3x^2-6x+3)}{(x^3-3x^2+3x)^2}$

$0 = 3x^2-6x+3$

$0 = x^2-2x+1$

$0 = (x-1)^2$

$x = 1$

$y = \dfrac{3}{1(1-3+3)}$

$= 3$

$\therefore (1,3)$

but $\left.\dfrac{d^2y}{dx^2}\right|_{x=1} = 0$ (CAS)

$\therefore (1,3)$ is a H.P.O.I

As $x \to \infty, y \to 0^+$

As $x \to -\infty, y \to 0^-$

$\left[\lim_{x\to\infty} \dfrac{3}{x^3}\right]$.

Q31. $y = \dfrac{x^3}{x-2}$

y int: $(0,0) \Rightarrow$ x-int.

vertical: $x \neq 2$

$y = \dfrac{x^2(x-2)+2x(x-2)+4(x-2)+8}{x-2}$

$= x^2+2x+4+\dfrac{8}{x-2}$

As $x \to \infty$, $y \to (x^2+2x+4)^+$

As $x \to -\infty$, $y \to (x^2+2x+4)^-$

parabolic asymptote: $y \neq x^2+2x+4$

Using CAS, $\dfrac{dy}{dx} = 0$ at $(3, 27)$

$\dfrac{dy}{dx} = 0$ and $\dfrac{d^2y}{dx^2} = 0$ at $(0,0)$.